KOMPANEETS MODEL FOR A THERMAL WAVE

G. V. Fedorovich

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The velocity of the leading edge of a thermal (heat) wave increases exponentially if the density of the gas in front of the leading edge of the wave falls in accordance with a similar law. When the wave propagates in a nonuniform atmosphere the shape of the leading edge may deviate from spherical and ultimately the thermal wave may "break through" the atmosphere.

The fact that the shock wave arising from a strong explosion might break through the atmosphere was predicted by Kompaneets [1]; this effect is due to the exponential increase in the velocity of the leading edge of the shock wave in traveling upward. By comparison with this increase, the slow (power-law) change in velocity, associated with the reduction in the specific energy of the gas with increasing volume of the shock wave, becomes insignificant. An analogous effect may occur in the development of the thermal wave which in a severe explosion precedes the shock wave [2]. It follows from consideration of a spherical thermal wave that the velocity v of its leading edge is determined by the thermal diffusivity $\chi = aT^n$ close to the leading edge (T is the volume-average gas temperature) and the volume of the wave

$$v = \chi V^{-1/3} \tag{1}$$

If, following [3], we regard the specific heat of the gas C as constant (this restriction is not fundamental) the thermal diffusivity χ is determined in terms of the density of the gas and the Rosseland range of light l in the following way: $\chi = 16\sigma T^3 l/{}^3\rho C$ (σ is the Stefan–Boltzmann constant), i.e., it varies inversely proportionally to a certain power of the gas density. This also applies to the coefficient a in the expression approximating the dependence of χ on T. In an exponential atmosphere in which $\rho \sim \exp[-z/Z]$ the values of χ and a increase exponentially with the height z; however, the characteristic scale of the changes in these quantities Z_0 is smaller than the scale of the changes in air density Z. Hence, according to Eq. (1), in an inhomogeneous atmosphere the leading edge of the thermal wave travels upward more rapidly than downward, and its shape deviates from spherical.

As in the case of a shock wave, a "pulling-out" effect at the leading edge may occur for a thermal wave formed at a height at which its limiting radius R* (corresponding to the transition of a thermal into a shock wave) is comparable with the scale of the change in the function $a(z)$. The value of R* may be estimated by equating the rate of energy transfer through thermal radiation $\sigma T^4/\rho_0 CT$ (ρ_0 is the density of the gas at the height of energy release) to the velocity of hydrodynamic motion, which is proportional to the velocity of sound, in order of magnitude equal to $\sqrt{CT}$. Since $T \approx \varepsilon/\rho_0 CV$ (E is the total energy released), at the instant of the transition of the thermal wave into a shock wave

$$R^* \approx V^{1/3} \approx E^{1/3}\sigma^{2/15}\rho_0^{-7/15}C^{-8/15} \tag{2}$$

If in accordance with [2] we assume that for $\rho_0 \approx 10^{-3}$ g/cm^3 and $E \approx 10^{21}$ erg the value of R* is approximately 14 m, it follows from Eq. (2) that for the same value of E $R \approx Z_0 \approx 3$ km if $\rho_0 \approx 10^{-8}$ g/cm^3, which occurs at a height of about 70 km.

Let us consider the shape of the leading edge of a thermal wave, formed at a height at which $R > Z_0$, in cylindrical coordinates, having their origin at the point of energy release. We use R(z, t) to denote the coordinate of the leading edge, z_2 and z_1 the coordinates of the upper and lower points of the leading edge respectively. The volume of the gas behind the leading edge of the thermal wave is

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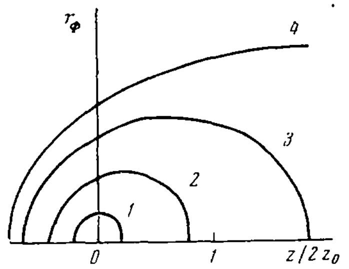

Fig. 1

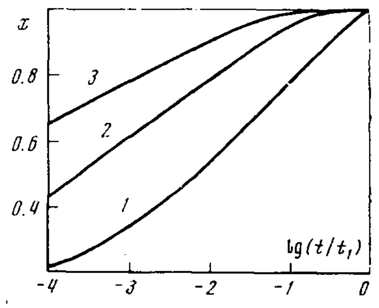

Fig. 2

$$V(t) = \pi \int_{z_1(t)}^{z_2(t)} R^2(z, t)\, dz \tag{3}$$

The mean gas temperature may be determined by using the law of conservation of thermal energy

$$E = CT \int_V \rho\, dV$$

which gives

$$T(t) = \frac{E}{\pi C \rho_0} \left\{ \int_{z_1(t)}^{z_2(t)} R^2(z, t) \exp\{-z/Z\}\, dz \right\}^{-1} \tag{4}$$

If a varies with height z according to the equation $a = a_0 \exp[-z/Z_0]$, the equation for R(z, t) coincides with the analogous equation for the leading edge of the shock wave derived in [1]

$$\left(\frac{\partial R}{\partial x}\right)^2 = Z_0^2 \exp\left[2\frac{z}{Z_0}\right]\left[1 + \left(\frac{\partial R}{\partial z}\right)^2\right] \tag{5}$$

where in contrast to [1]

$$x = \frac{a_0}{Z_0} \int_0^t T^n(t')\, V^{-1/3}(t')\, dt' \tag{6}$$

The solution of Eq. (5) has the same form as that derived in [1]

$$R(z, t) = Z_0 \arccos\left\{\frac{1 - x^2}{2} \exp\left(\frac{z}{z_0}\right) + \frac{1}{2} \exp\left(-\frac{z}{Z_0}\right)\right\} \tag{7}$$

The difference between the laws of motion of the leading edges of the thermal and shock waves is due to the difference in the t = t(x) relationships. For the thermal wave

$$\begin{aligned} t(x) &= \frac{Z_0^2}{a_0}\left(\frac{\pi}{2}\right)^{1/3}\left(\frac{\pi\rho_0 C Z_0^3}{2E}\right)^n \int_0^x dx' F^{1/3}(x')\, \Phi^n(x') \\ F(x) &= \int_{+2\ln(1+x)}^{-2\ln(1-x)} du \arccos^2\left[\frac{1-x^2}{2}\exp\left(\frac{u}{2}\right) + \frac{1}{2}\exp\left(-\frac{u}{2}\right)\right] \\ \Phi(x) &= \int_{-2\ln(1+x)}^{-2\ln(1-x)} du \exp\left(-\frac{Z_0}{Z}u\right)\arccos^2\left[\frac{1-x^2}{2}\exp\left(-\frac{u}{2}\right) + \frac{1}{2}\exp\left(-\frac{u}{2}\right)\right] \end{aligned} \tag{8}$$

The time required to break through the atmosphere $t_1 = t(x = 1)$ depends more strongly on the parameters of energy release than in the case of the shock wave; with increasing energy E it falls in accordance with the law $t_1 \sim E^{-n}$, whereas for a shock wave $t_1 \sim E^{-1/2}$. This also applies to the dependence of t_1 on the air density ρ_0. Since according to [1] the deviation of the shock wave from spherical form becomes appreciable for $t \approx t_1/6$, and that of the thermal wave for $t \approx 10^{-2} t_1$ (if n = 3), we may reasonably assert that the thermal wave departs from spherical form more rapidly than the shock wave.

Figure 1 (curves 1, 2, 3, 4) shows the profiles of the leading edge of the thermal wave [curves R(z)] at the instants corresponding to values of the dimensionless parameter x = 0.2, 0.5, 0.8, and 1. The dependence of x on time t differs for different n. The x = x(t) curves calculated with the help of Eq. (8) for n = 3, 5, 7 (curves 1, 2, 3 respectively) are presented in Fig. 2. The set of data presented in Figs. 1 and 2 enable us to determine the profile of the thermal wave at various moments of time.

LITERATURE CITED

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